Research Statement

My research lies in the field of algebraic topology, and to some extent also in representation theory and mathematical physics. In particular, I am interested in a type of cohomology theory associated to elliptic curves, called elliptic cohomology. Although strictly speaking it is an object in algebraic topology, elliptic cohomology has strong relationships to number theory, quantum field theory and geometric reprentation theory. It is its connections to these other areas which I think makes elliptic cohomology so interesting.

Past work

In my PhD thesis, I constructed a complex analytic, torus-equivariant version $\mathcal{E}\ell\ell_T$ of elliptic cohomology, along the lines conjectured by Charles Rezk in his paper *Looijenga line bundles in complex analytic elliptic cohomology* (arXiv:1608.03548, see Section 5.2). The cohomology theory $\mathcal{E}\ell\ell_T$ is defined on the category of torus-equivariant, finite CW-complexes, and takes values in coherent holomorphic sheaves on the moduli stack of complex elliptic curves. In my thesis, I also established the relationship between $\mathcal{E}\ell\ell_T$ and Nitu Kitchloo's construction of elliptic cohomology $\mathcal{K}_{LT}$, which he constructs from objects in conformal field theory. More precisely, Kitchloo actually defines a G-equivariant cohomology theory ${}^{\alpha}\mathcal{K}_{LG}$ for simple, simply-connected compact Lie groups G, and twisted by an element $\alpha \in H^4(BG : \mathbb{Z})$. So, I first construct an untwisted, torus-equivariant version $\mathcal{K}_{LT}$ of Kitchloo's theory, and then I construct a type of character map from $\mathcal{K}_{LT}$ to $\mathcal{E}\ell\ell_T$ in order to compare the two theories. Kitchloo's construction may be found in his paper *Quantization of the modular functor and equivariant elliptic cohomology* (arXiv:1407.6698).

Future work

There are three projects that I plan to continue working on in the future.

1. **Twisted equivariant, complex analytic elliptic cohomology.** (Joint with Dan Berwick-Evans.) Let T be a compact torus, and let X be a finite T-CW complex. Given a class $\alpha \in H^4(X \times_T ET; \mathbb{Z})$, I would like to construct an α-twisted version of $\mathcal{E}\ell\ell_T(X)$. The first step in such a construction is to build an object $\widetilde{[X/T]}$ over the quotient stack $[X/T]$ from the data of a cocycle representing α. There are certain cases which point to how the object $\widetilde{[X/T]}$ might be constructed in general. For example, in the non-equivariant picture, suppose that X is a manifold with spin structure, so that X comes equipped with a map $\phi : X \to [pt/Spin(n)]$. In 2012, Schommer-Pries modelled $String(n)$ as a 2-group extension of $Spin(n)$. Using this, one may define a 2-gerbe $\tilde{X}$ over X as the pullback of $[pt/String(n)]$ along ϕ, corresponding to the pullback of the canonical class in $H^4(BSpin(n); \mathbb{Z}) \cong \mathbb{Z}$.

In the equivariant case, we assume that X is equipped with a T-equivariant spin structure. We would then hope to similarly construct an object $\widetilde{[X/T]}$ over $[X/T]$, and input this into the construction $\mathcal{E}\ell\ell_T$ from my thesis.

2. **Power operations in complex-analytic, equivariant elliptic cohomology.** I would like to give a concrete description of a theory of power operations for $\mathcal{E}\ell\ell_T$. In the process of constructing $\mathcal{E}\ell\ell_T(X)$, we find that X actually determines a $\mathbb{C}^\times \times (\mathrm{SL}_2(\mathbb{Z}) \ltimes \check{T}^2)$-equivariant sheaf on a complex analytic space

$$L \times (\mathrm{Lie}(T) \otimes \mathbb{C}).$$

Here, $\check{T}$ is the cocharacter lattice of T and L is the space of based lattices in $\mathbb{C}$. The action of $\mathrm{SL}_2(\mathbb{Z})$ on L extends to an action of the monoid M of 2×2 integral matrices with nonzero determinant, and there is a scalar action of $\mathbb{C}^\times$ which commutes with M. The combined action yields source and target maps

$$L \quad \xleftarrow{s} \quad \mathbb{C}^\times \times M \times L \quad \xrightarrow{t} \quad L$$

corresponding to the projection and the action respectively. Geometrically, these are the source and target maps of the category of "elliptic curves and isogenies". It is known that power operations in elliptic cohomology correspond to isogenies of the underlying elliptic curve. Extending the action of $\mathrm{SL}_2(\mathbb{Z})$ to an action of M should therefore lead to the construction of power operations for $\mathcal{E}\ell\ell_T$.

3. **Twisted quasi-elliptic cohomology and an elliptic character map.** (Joint with Zhen Huan.) In her thesis, supervised by Charles Rezk, Zhen Huan constructed a cohomology theory called quasi-elliptic cohomology, and developed a theory of power operations for it. Zhen and I are now in the advanced stages of constructing a twisted version of quasi-elliptic cohomology, and an 'elliptic character map' from this into Devoto's twisted elliptic cohomology. In future papers, we hope to apply this map to compare power operations in the two theories.